Optimization of Design of Column-Reinforced Foundations

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Abstract: The design of foundations on soft ground reinforced by columns usually involves two important verifications, namely, checking for adequate bearing capacity and checking for acceptable settlement performance. This paper details a comprehensive methodology for determining the optimized portion of the ground area that should be improved by the installation of columns. The optimization is required to avoid an overly conservative design and, consequently, the use of uneconomical quantities of material to construct the columnar reinforcement. The basis of the suggested methodology consists of first estimating the minimum improvement area ratio (IAR) required to ensure attainment of the required design bearing capacity of the reinforced soil and then determining an upper-bound or maximum value of IAR by considering the issue of allowable settlement. Optimization is then performed on the IAR within the range defined by these bearing capacity and settlement limits. Analysis of three case studies provides an illustration of the implementation of this novel design methodology, which has been incorporated into software recently developed to assist in the design of soil foundations reinforced by columns and to provide cost-effective solutions for this type of foundation. **DOI: 10.1061/(ASCE)GM.1943-5622.0000384.** *© 2014 American Society of Civil Engineers.*

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Introduction

Reinforcement of weak soils by stiffer columns has numerous benefits, including the potential to increase the bearing capacity, reduce settlements under working loads, and accelerate consolidation of the soft soil by acting as a drainage pathway (whenever free-draining material is used to construct the columns). The cost of most schemes involving column-reinforced foundations (CRFs) constructed using stone columns, compaction piles, or deep soil-mixing techniques is essentially controlled by the volumetric fraction of material introduced into the host soil (or native ground). This fraction is referred to as the improvement area ratio (IAR), which is defined by the total cross-sectional area of the columns divided by the gross area of the loaded foundation.

Weak soils include highly compressible clays with an undrained shear strength of less than about 30 kPa, Young's modulus lower than about 2 MPa, and loose sands having a friction angle of less than about 30°, i.e., a standard penetration test (SPT) blow count of less than 10 (Bergado et al. 1996).

Depending on the adopted technique of column reinforcement, the value of IAR generally falls within the following ranges (Bergado et al. 1996):

- 0.15–0.35 for stone columns, for which the strength of column material is mainly characterized by a high friction angle (i.e., >40°);
- 0.15–0.7 for deep mixing, where the strength of the column material is mainly characterized by enhanced cohesive strength (usually at least 20 times more than that of the host soil); and
- 0.05–0.15 for vibrocompaction, conducted with or without added material, for which the strength of the column material is usually characterized by a moderate apparent cohesion and enhanced friction angle.

The design of foundations on ground reinforced by columns usually requires two important verifications, namely, checking for adequate bearing capacity and checking for acceptable settlement performance. The design also can involve consideration of the acceleration of any consolidation settlement of the host soil when the columns behave like vertical drains, as well as the liquefaction potential of the host soil, which is relevant mostly for loose saturated sands. Prior methods suggested for the design of column-reinforced foundations (CRFs) are mainly based on the unit-cell model (UCM), which provides the particular advantage of allowing validation of analytical and numerical predictions with experimental records. The latter usually have been obtained from triaxial tests (e.g., Barksdale and Bachus 1983; Ghionna and Jamiolkowski 1981; Bouassida 1996; Poorooshasb and Meyerhof 1997; Ambily and Gandhi 2007). The UCM has been used to derive homogenized deformation and strength characteristics of CRFs (Bouassida et al. 1995) and to predict the yield stress of the reinforced soil (Jellali et al. 2005), as well as to predict its elastoplastic behavior (Abdelkrim and de Buhan 2007). The UCM is conceived from the distribution of a group of columns installed in a regular pattern. Geometrically, it is a reproducible volume of reinforced soil that includes one column. For example, columns installed in square and triangular grid patterns correspond, respectively, to parallelepiped and hexagonal cylinders as periodic volumes of unit (or composite) cell models. Then, to carry out the well-known axisymmetric model for bearing capacity and settlement calculation, an equivalent cylindrical unit cell having a circular cross section is adopted (Balaam and Booker 1981). The UCM assumes that lateral deformation at the boundary of the UCM is zero in conformity with oedometer conditions. The main advantage of this laboratory model is the assessment of theoretical results in regard to bearing capacity, settlement, and acceleration of consolidation of a compressible soil reinforced by drained column material. Accordingly, Bouassida (1996), Ambily and Gandhi (2007), and others conducted experimental investigations adopting the UCM to study the behavior of soil reinforced by columns.

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The isolated column model (ICM) is composed of a single inclusion surrounded by unlimited initial soil volume. It also has been used to predict the ultimate bearing capacity and settlement of CRFs, and these predictions have been compared with data recorded from full-scale loading tests, usually performed in field trials conducted prior to installation of most of the columns (Bergado and Lam 1987; El Ghabi et al. 2010). The ICM is therefore a useful tool for the validation of in situ conditions. It also assumes that the surface loading is applied directly to the column only, so effectively it can be considered to be a special case of full soil reinforcement, i.e., a full-replacement configuration (IAR = 100%). This severe limitation of the ICM means that the influence of the IAR on the performance of CRFs cannot be studied using this approach. It also means that use of the ICM, as adopted in the French method [French Committee for Soil Mechanics and Foundations (CFMS) 2011], generally leads to an overestimation of the ultimate bearing capacity of stone column foundations compared with predictions given by other methods (Ellouze et al. 2010). Accordingly, adopting three-dimensional (3D) modeling of CRFs would seem more appropriate to obtain representative predictions of the role of the IAR as a key parameter in design.

It is also worth noting that the existing methods of design of CRFs are often aimed at obtaining a unique verification of either the bearing capacity (less likely) or the settlement (more likely) as the controlling factor in design. As such, these methods of design have been formulated for a unique technique of column installation, e.g., stone columns (Priebe 1995; CFMS 2011) or the deep soil-mixing method (Kitazume et al. 1996; Broms 2000). In these contributions, optimization of the quantity of required column material was not a matter of first importance. Therefore, values of the IAR were usually dictated by experience gained from existing CRF projects, with the main focus of the proposed design methods being on the reduction of settlement and perhaps a minor interest in the acceleration of that settlement. The latter is mainly related to stone column reinforcement.

In this paper, a comprehensive methodology for the design of CRFs is suggested that satisfies the requirements of both bearing capacity and settlement. The suggested methodology is based on research results that have been obtained in rigorous theoretical frameworks involving 3D modeling of CRFs as end-bearing and floating columns. In this approach, the constituents of a CRF, i.e., the weak native soil, and the reinforcing columns are assumed to be different homogeneous and isotropic materials. First, limit analysis is used to determine the ultimate bearing capacity of the reinforced soil by considering the strength criteria of the initial soil and the column material, with both obeying the rigid perfect plastic Mohr-Coulomb model (Bouassida et al. 1995). Second, the variational method in linear elasticity is used to predict the settlement of the reinforced soil (Bouassida et al. 2003a).

The suggested methodology requires three steps to derive an optimized value of IAR for any given situation. The IAR was found to be the key parameter for the design because the reinforced ground is considered by a group of columns. Once the optimized IAR is determined for a given column technique, the diameter of the inclusions is provided within a certain range (e.g., for stone columns it is between 0.8 and 1.2 m), and the spacing between columns is then deduced for a chosen grid pattern. This design procedure is already programmed in *Columns 1.0* software (Bouassida and Hazzar 2012).

The reductions in project costs achieved by adopting the suggested methodology of design are evidenced by various cases histories: a tank foundation resting on soft clay reinforced by end-bearing stone columns, an embankment founded on soft clay reinforced by either floating lime-cement-treated columns or by full substitution of the host soil either by compacted sand or cement-stabilized host soil, and an embankment founded on columns constructed using the deep-mixing method (DMM).

Problem Statement

The first stage in the design process of a CRF involves verification of bearing capacity, which constitutes a necessary condition for the stability of the CRF. Thereafter follows verification of the settlement criteria for the CRF, which is also a necessary condition that must be satisfied to achieve an adequate design. Reinforcement of the host soil by columns is achieved by vertical inclusions, either end bearing or floating, of length H_c installed under the loaded area A, referred to as the Foundation in Fig. 1. The locations of the columns having circular cross sections can be arbitrary, and so may be their diameters. The total cross-sectional area of the columns is denoted as A_c. The IAR, denoted by the symbol η, is then given simply as

$$\eta = \frac{A_c}{A} \tag{1}$$

Verification of bearing capacity involves estimation of the unit weights and shear-strength characteristics of the component soils, i.e., friction angle and undrained shear strength of the initial host soil and column material. The settlement is estimated assuming linear elastic material response using Young's moduli and Poisson's ratios of the column material (E_c and ν_c) and the weaker host soil (E_s and ν_s). Because introduction of the column material is intended to reinforce the host soil and thus increase its overall effective stiffness, it generally follows that $E_c > E_s$.

The first step of the methodology is to consider lower and upper bounds on the ultimate bearing capacity of the CRF, obtained by limit-analysis approaches of plasticity theory, to determine the minimum value of IAR ($\eta_{\min}$) that complies with the maximum allowable bearing resistance of the CRF (the design capacity).

In the second step, the settlement of the CRF is estimated within the framework of linear elasticity. Application of a variational principle enables an assessment of the lower bound of the apparent Young's modulus that is associated with the maximum allowable settlement. Given a specified maximum allowable settlement of the CRF, an upper bound or maximum value of IAR ($\eta_{\max}$) is then identified.

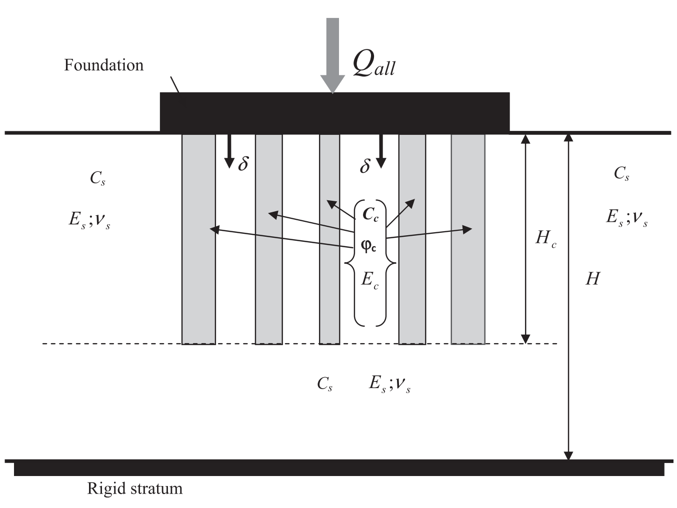

Fig. 1. Modeling reinforced soil by floating columns

Finally, the value of IAR (η) is then optimized within the range defined by these first two steps ($\eta_{\min} \leq \eta \leq \eta_{\max}$) by considering the specified settlement criteria and a selection of various models for predicting the settlement of the CRF. In this way, an optimal value of IAR (η_{opt}) is determined that does not overestimate the quantity of reinforcing column material.

The methodology to be detailed in the following sections applies for different types of structures (e.g., rigid rafts or footings and embankments) for which the settlement at the surface of the reinforced soil δ is assumed to be uniform.

The proposed design methodology is then based on the verifications of bearing capacity and settlement that are conducted for the short-term condition that is the first requirement for the stability of a CRF. Once the optimized area ratio is identified, it should be checked whether it fits within the prescribed ranges given earlier. If so, a finite-element (FE) procedure can be used to predict the long-term behavior of the structure founded on the reinforced columnar foundation.

Bearing Capacity of CRF

The limit-analysis framework is usually considered by performing a static analysis to derive a lower bound on the ultimate bearing capacity of the CRF and a kinematic analysis to determine its upper bound. Based on existing results, which have been obtained by Bouassida et al. (1995) and Bouassida and Porbaha (2004), predictions of the minimum values of IAR are detailed in subsequent paragraphs.

Considering that the weak soil is almost purely cohesive, the behavior of the column material is either frictional, such as that of stone, gravel, or sand (for which cohesion can usually be neglected), or purely cohesive, such as soft clay treated by cement, lime, or a combination of the two. Hence, for calculation purposes, two main categories of column material, i.e., cohesive-frictional and purely cohesive, are used to study the most used techniques, which are stone columns and deep mixing.

Stone Columns

By adopting the static approach of limit analysis, a lower bound on the ultimate bearing capacity of the CRF, Q_{ult}^{-}, is determined for a purely cohesive soil reinforced by a group of end-bearing columns made up of cohesive-frictional column material. This lower-bound solution obtained by Bouassida et al. (1995) may be written as

$$Q_{\mathrm{ult}}^{-}/A = \sigma_{\mathrm{ult},rs} = (1-\eta)\sigma_{\mathrm{ult},s} + \eta\sigma_{\mathrm{ult},c} \tag{2}$$

In Eq. (2), the vertical stresses induced within the column material and the host soil, respectively, i.e., $\sigma_{\mathrm{ult},c}$ and $\sigma_{\mathrm{ult},s}$, may be expressed in terms of the friction angle φ_c and undrained shear strength C_c of the column material and the undrained shear strength of the host soil C_s. Explicit expressions for the ultimate load Q_{ult}^{-} are provided in the Appendix for the case of a purely cohesive soil reinforced by cohesive-frictional column material. Explicit expressions for the individual terms $\sigma_{\mathrm{ult},c}$ and $\sigma_{\mathrm{ult},s}$, can be easily derived from this detail.

An appropriate value of the factor of safety against yielding, denoted F_{rs}, is adopted, and regardless of the failure mode of the reinforced soil, the allowable bearing capacity of the CRF, denoted as $\sigma_{\mathrm{all},rs}$, is defined by

$$\sigma_{\mathrm{all},rs} = \frac{(1-\eta)\sigma_{\mathrm{ult},s} + \eta\sigma_{\mathrm{ult},c}}{F_{rs}} \tag{3}$$

As a first approximation, the predicted lower bound on the bearing capacity usually can be associated with a factor of safety within the range $1 < F_{rs} < 2$. The allowable bearing capacity of the CRF, compared with the mean vertical stress induced by the load applied to the foundation, denoted Q_{actual}, should comply with the necessary condition

$$Q_{\mathrm{actual}}/A \leq \sigma_{\mathrm{all},rs} \tag{4}$$

Combining Eqs. (3) and (4), the minimum value of IAR, denoted as $\eta_{\min}$, is then identified so that

$$\eta \geq \frac{F_{rs}(Q_{\mathrm{actual}}/A) - \sigma_{\mathrm{utl},s}}{\sigma_{\mathrm{utl},c} - \sigma_{\mathrm{utl},s}} = \eta_{\min} \tag{5}$$

As can be observed from Eq. (5), the minimum value of IAR, $\eta_{\min}$, corresponds to the minimum quantity of column material to be incorporated in the host soil such that the ultimate bearing capacity increases from $\sigma_{\mathrm{ult},s}$ to $\sigma_{\mathrm{ult},rs}$. Otherwise, if $\eta_{\min} \leq 0$, the reinforcement is unnecessary because the allowable bearing capacity of the unreinforced soil is sufficient to withstand the applied load exerted by the foundation.

Deep-Mixing Columns

In practice, a CRF constructed by the DMM can be modeled reasonably as a relatively uniform clay medium with an undrained shear strength C_s reinforced by another purely cohesive (claylike) column material with undrained shear strength C_c (Kitazume et al. 1996). This case of reinforcement by a group of end-bearing columns has been investigated by performing limit analysis using the kinematic approach, from which an upper-bound solution of the ultimate bearing capacity was proposed. The assessment of this upper-bound solution was found to be in good agreement with experimental data recorded from scale models. Considering the same unit weight for the soft clay and the cemented column material, it can be shown that the upper bound of the ultimate bearing capacity of a CRF may be expressed as (Bouassida and Porbaha 2004)

$$\frac{Q_{\mathrm{ult}}^{+}}{C_s A} = 2\left\{\sqrt{2} + \sqrt{[1+\eta(K_c-1)][2+\eta(K_c-1)]}\right\} \tag{6}$$

where $K_c = C_c/C_s$ is the ratio of the undrained shear strength of the column material to that of the host soft clay. Current experience from the practice of deep mixing indicates that the value of K_c varies over a large range, typically from 10 to 130, whereas the value of IAR usually ranges from 0.1 to 0.7 (Broms 2000). Within these margins for K_c and η, it can be shown that the upper bound given by Eq. (6) can be approximated by the following linear relationship (Bouassida and Porbaha 2004):

$$\frac{Q_{\mathrm{ult}}^{+}}{C_s A} = 4\sqrt{2} + 2\eta(K_c - 1) \tag{7}$$

The upper-bound solution of bearing capacity factor (BCF) given by Eq. (6), within the margins $0.1 \leq \eta \leq 0.7$ and $10 \leq K_c \leq 130$, is approximated by the relationship given by Eq. (7). It can be verified that the relative error between the functions given by Eqs. (6) and (7) is less than 1%, which confirms the usefulness of Eq. (7), from which the minimum IAR was derived. Therefore, Eq. (7) can be written in the form

$$\sigma_{\mathrm{ult},rs}^{+} = \sigma_{\mathrm{ult},s}^{+} + \eta\sigma_{\mathrm{ult},c}^{+} \tag{8}$$

where $\sigma_{\mathrm{ult},c}^{+}$ and $\sigma_{\mathrm{ult},s}^{+}$ = ultimate vertical stress components that are derived from the upper bound solution, as given by Eq. (7).

For the deep-mixing case of reinforcement by end-bearing columns, Bouassida and Porbaha (2004) have suggested a close bounding between the upper- and lower-bound solutions. The lower-bound solution for the deep-mixing case is expressed in the same form as obtained for the upper-bound solution. In fact, the lower-bound solution is written as $q^{-}_{\mathrm{ult},r} = 4 + 2\eta(K_c - 1)$ in a form that is identical to that given by Eq. (7). Therefore, identification of the minimum IAR using the lower-bound solution is quite similar to that obtained when the upper-bound solution is adopted. Because the relative error between the lower and upper bounds of the ultimate bearing capacity is 10%, as validated experimentally by Bouassida and Porbaha (2004), the difference in minimum IAR will not be significant when considering either a lower-bound or an upper-bound solution.

Following the same steps as those followed from Eq. (3) through Eq. (5) for the lower-bound solution, and considering the upper bound given by Eq. (8), the minimum acceptable value of IAR may be written as

$$\eta \geq \frac{F_{rs}(Q_{\mathrm{actual}}/A) - \sigma^{+}_{\mathrm{utl},s}}{\sigma^{+}_{\mathrm{ult},c}} = \eta^{+}_{\min} \tag{9}$$

Determination of the bearing capacity of a CRF, constructed using floating columns, has been studied by Bouassida et al. (2009). The results obtained indicate that the lower-bound solutions for the ultimate bearing capacity of a CRF constructed with end-bearing columns are also applicable to floating columns. However, there are limitations on the length of the columns for this equivalence to apply, and those limitations depend on the characteristics of the soil. However, as demonstrated by Bouassida et al. (2009), these requirements are achieved in most, if not all, practical cases. Therefore, the prediction of the minimum IAR provided previously remains valid for cases where the CRF is constructed using floating columns (Bouassida et al. 2009).

Settlement of CRF

End-Bearing Columns ($H = H_c$)

When $H = H_c$ (Fig. 1), the case corresponds to reinforcement by end-bearing columns, for which the total settlement equals that of the reinforced soil mass $\delta_{\mathrm{tot}} = \delta_r$.

The apparent Young's modulus of a CRF, denoted as E_{rs}, introduced as

$$E^{-}_{rs} \leq E_{rs} = \frac{Q_{\mathrm{actual}}/A}{\delta_r/H_c} \tag{10}$$

can be approximated using the variational methods in linear elasticity. In particular, Bouassida et al. (2003a) used the principle of minimum complementary energy, which simply states that for all elastic stress states satisfying the boundary conditions, those that also satisfy the equilibrium equations make the complementary energy a local minimum. Applying this principle to a rigid foundation of area A subjected to an allowable working load Q_{actual}, which induces an assumed uniform settlement of the CRF, denoted by δ_r, Bouassida et al. (2003a) established the expression of a lower bound of the apparent Young's modulus of a CRF, denoted as E^{-}_{rs}, that is greater than the so-called homogenized Young's modulus, i.e., $E^{-}_{rs} \geq (1-\eta)E_s + \eta E_c$. Accordingly, from Eq. (10), a more conservative prediction of the settlement of a CRF, δ_r, is obtained (Bouassida 2013)

$$\delta_r \leq \frac{(Q_{\mathrm{actual}}/A)H_c}{(1-\eta)E_s + \eta E_c} = \delta^{+}_r \tag{11}$$

The upper-bound estimate of settlement δ^{+}_r in Eq. (11) has been obtained by considering the conservative homogenized Young's modulus of the CRF. It is then required that the allowable settlement of the CRF, denoted as $\overline{\delta}_r$, should comply with δ^{+}_r so that

$$\overline{\delta}_r \leq \delta^{+}_r \tag{12}$$

At this stage of the design procedure, as a first check on the settlement of the CRF, Eq. (11) is used to verify whether the minimum value of IAR, calculated either from Eq. (5) or from Eq. (9), complies with the prescribed allowable settlement of the CRF. If so, then the predicted minimum value of IAR, $\eta_{\min}$, can be accepted for the final design because both bearing capacity and settlement requirements have been verified. It is noted that this situation is most likely to be encountered in cases involving the improvement of loose sands using the vibrocompaction method. The bearing capacity of loose sands having a friction angle close to 29° is adequate even when the soil is not reinforced. Therefore, the optimized IAR is only relevant to the verification of allowable settlement. Accordingly, the optimized IAR reduces to that of maximum IAR.

In cases where the minimum value of IAR does not satisfy the allowable settlement criterion, the value of IAR must be increased such that $\eta > \eta_{\min}$. This situation usually arises in cases where soft clay is reinforced by stone columns. For different case histories, Bouassida and Hazzar (2012) showed that the minimum IAR does not satisfy the required settlement criterion of soft clays reinforced by stone columns. It was found that the optimized IAR is often higher than the minimum IAR. In fact, because of the high compressibility of soft clays, even when reinforced by end-bearing columns, the allowable settlement is always satisfied only with a greater IAR than the minimum one.

In such cases, the design then proceeds by combining Eqs. (11) and (12) to obtain

$$\eta \leq \frac{(Q_{\mathrm{actual}}/A)\left(H_c/\overline{\delta}_r\right) - E_s}{E_c - E_s} = \eta_{\max} \tag{13}$$

From Eq. (13), a maximum value of IAR, $\eta_{\max}$, is then identified. This value indicates the limit beyond which the volume of reinforcement material definitely will be overestimated because a truly conservative value of Young's modulus of the CRF has been considered for the settlement estimation, as indicated in Eq. (11).

Eq. (13) shows that the maximum IAR essentially depends on the allowable settlement of the reinforced layers and obviously on the lengths of the columns. Therefore, for end-bearing or floating columns, the maximum IAR remains unchanged. This parameter does not depend on the thickness of the unreinforced layer, for which the long-term settlement, especially in the case of highly compressible layers, also should be allowable. However, in floating columns, a smaller allowable settlement is used in the calculations because the settlement of the unreinforced layer is deducted from the total allowable settlement.

Eqs. (5), (9), and (13) provide bounds on the optimal value of IAR, i.e.

$$\eta_{\min} \leq \eta \leq \eta_{\max} \tag{14}$$

Eq. (14) provides the range of values of IAR that satisfy both the allowable bearing capacity and settlement criteria for the CRF. Within this range, the optimized value of IAR, η_{opt}, may then be determined as a function of the allowable settlement adopted for design.

An iterative procedure is then carried out within the range $[\eta_{\min} : \eta_{\max}]$. This is achieved by incrementing η in suitably small steps within the defined range, beginning at the minimum value $\eta_{\min}$ and then predicting the settlement corresponding to each particular value of η.

The suggested algorithm (captured in the *Columns 1.0* software, to be subsequently described) used to perform this iteration has a number of different methods programmed into it by which the settlement may be estimated. These are the methods proposed by Balaam and Booker (1981), Chow (1996), Bouassida et al. (2003a), and CFMS (2011). In each case, the predicted settlement is compared with the adopted allowable settlement. The optimal value of IAR (η_{opt}) is captured when the predicted settlement is less than or equal to the specified allowable settlement. Users have the option of deciding which of the settlement prediction methods they prefer to adopt for this purpose. This decision often will be linked to the particular construction method adopted for the CRF.

Floating Columns ($H > H_c$)

Because it has been assumed that the soil behaves as a linear elastic continuum for the purpose of settlement analysis, then the total settlement is the sum of the settlement component of the CRF soil mass, i.e., δ_r, and the settlement of the unreinforced underlying layer(s), δ_{ur}. The settlement component δ_r of the reinforced soil extending from the surface to a depth H_c is estimated using the method detailed earlier for end-bearing columns.

However, for the case of floating columns, it is necessary to verify that the total settlement $\delta_{\text{tot}} = \delta_{rs} + \delta_{ur}$ is also within allowable limits. This verification is especially important when the unreinforced underlying layers are compressible saturated clays, in which case an estimate of the long-term consolidation settlement is also required. In such cases, the settlement of the unreinforced soil layers can be predicted by application of Terzaghi's theory of one-dimensional consolidation.

Further, a lower-bound estimate of the settlement of a CRF may be derived using the theory of elasticity, and in particular, it may be expressed in terms of the homogenized modulus $E_{\text{oedom},rs}$ of the reinforced ground (Bouassida et al. 2003b), defined by

$$E_{\text{oedom},rs} = (1-\eta)E_{\text{oedom},s} + \eta E_{\text{oedom},c} \tag{15}$$

where

$$E_{\text{oedom},i} = \frac{E_i(1-\nu_i)}{(1-2\nu_i)(1+\nu_i)} \quad i = s, c \tag{16}$$

and E_s, ν_s, E_c, and ν_c have been defined previously. The oedometer condition provides the highest apparent Young's modulus of the CRF, as expressed by Eq. (15). In some practical cases, especially where the dimensions of the loaded area are much larger than the thickness of the reinforced layer(s), e.g., tanks having large diameters, estimation of the oedometer settlement also may be of interest, as suggested by Chow (1996). Therefore, in this case, the appropriate lower-bound estimate of the settlement of a CRF can be written as

$$\delta_r^- = \frac{(Q_{\text{actual}}/A)H_c}{E_{\text{oedom},rs}} \tag{17}$$

The methodology of design detailed earlier has been incorporated recently into software known as *Columns 1.0* (Bouassida and Hazzar 2012). The optimization stage embodied in the algorithm of *Columns 1.0* is illustrated in Fig. 2. This software and its algorithm have been validated successfully using numerous case histories. In particular, the effectiveness of the design procedure in terms of the eventual cost of the final project has been demonstrated by recorded field data for a tank project founded on stone columns (Bouassida and Hazzar 2012).

Acceleration of Consolidation

When studying the reinforcement of soft clays by sand-compaction columns and stone columns made of free-draining material having an enhanced permeability, the potential acceleration of the consolidation of the soft clay should be addressed, in a second step, by considering the predicted optimized IAR, as detailed earlier during the preliminary design of a CRF. A poroelastic method has been implemented in *Columns 1.0* (Bouassida and Hazzar 2012) to predict the accelerated settlement of the CRF in such cases (Guetif and Bouassida 2005). This settlement, which depends in particular on IAR, represents the long-term settlement of the reinforced soil layer because the reinforcing columns also act as vertical drains.

It is also worth noting the improvement in the strength and deformation characteristics of the soft clay that result from its induced primary consolidation. Guetif et al. (2007) have reported results that

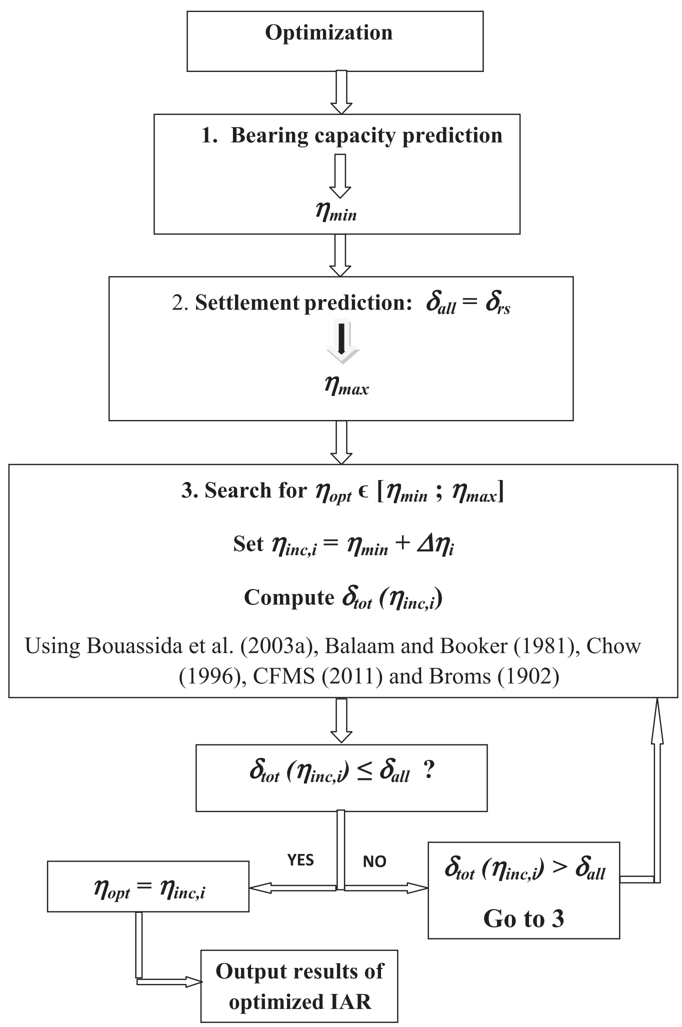

Fig. 2. Optimization of IAR by *Columns 1.0* software

indicate how Young's modulus of the soft clay is improved as a result of stone column installation and subsequent consolidation.

As the stiffness and strength of the initially soft soil is improved by the installation of columns, the overall bearing capacity of the CRF therefore should be increased beyond that which is predicted, assuming the original (unimproved) properties of the soft clay. Hence, use of the initial estimate of the undrained shear strength of the unimproved soil should guarantee a conservative estimate of the bearing capacity of the CRF.

Differential Settlements

Design methods for soils reinforced by columns usually assume that the settlement of a loaded foundation is uniform, as considered in Fig. 1. Such an assumption holds in the case of rigid foundations (e.g., raft footings) subjected to uniform surcharge. Further, the settlement at the surface of reinforced ground is uniform when a mattress layer, usually made of the same material as the installed columns (stone or sand), is spread out under the loaded foundation. In fact, such a mattress layer plays the same role when compressible soils are reinforced by vertical rigid inclusions (Boussetta et al. 2012).

However, significant differential settlements of a CRF are more likely to occur in cases where the reinforcement is provided by floating columns or in cases of nonuniformly distributed loading where the reinforcement is provided by end-bearing columns. In such cases, prediction of the detailed behavior of the CRF usually requires numerical analysis, e.g., using FE codes, on the basis of the optimized IAR determined by the methodology of design detailed in this paper. Indeed, numerical predictions also may help in adjusting the final length of the columns and their spacing.

Illustrative Case Histories

Oil Tank on End-Bearing Sand Compaction Columns (Tunisia)

An oil storage tank, 30 m in diameter, was built in the harbor area of La Goulette, a northern suburb of Tunis, Tunisia. The site was characterized by soft clay overlying firm sand. The tank loading was approximated by a quasi-uniform vertical stress of 80 kPa, which does not exceed the allowable bearing capacity of the soft clay layer. Nevertheless, the predicted settlement of the tank of about 40 cm greatly exceeds the allowable settlement of 10 cm. Therefore, reinforcement by 10-m-long end-bearing sand columns was adopted to reduce the final settlement and to accelerate significantly the consolidation settlement of the soft clay layer. Fig. 3 summarizes the geotechnical properties of the host soil and the column material.

Using the *Columns 1.0* software (Bouassida and Hazzar 2012), and adopting a maximum allowable settlement of the reinforced soil of 10 cm, the optimized IAR was estimated as 29.95%, and this could be achieved by installing 729 sand piles, 0.6 m in diameter, with axis-to-axis spacing of 1.06 m in a triangular mesh. The corresponding predicted allowable bearing capacity of the CRF has been verified as having a safety factor greater than 2, which was considered reasonable for a tank project where the tank has a diameter of 30 m and the soft clay is only 10 m thick.

Fig. 4 shows the evolution of settlement of the CFR as a function of the applied load for the optimized IAR. It is noted that the smallest settlements have been predicted by Chow's method because the horizontal component of deformation is neglected in this approach.

According to the study of a similar case history by Bouassida and Hazzar (2012) that considered evolution of the normalized apparent Young's modulus of the CRF as a function of the IAR, it was found that the settlement predicted by the method of Bouassida et al. (2003a) was conservative in comparison with predictions given by the three other methods. This is probably the result of use of 3D modeling without taking into account the improvement in the initial soil characteristics. Using the three other methods of settlement analysis, which are based on the UCM with or without assuming oedometer conditions, the settlement predictions are slightly underestimated.

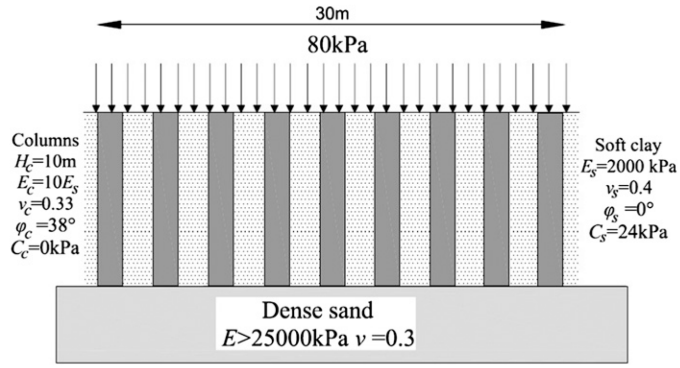

Fig. 3. Circular tank on end-bearing sand columns

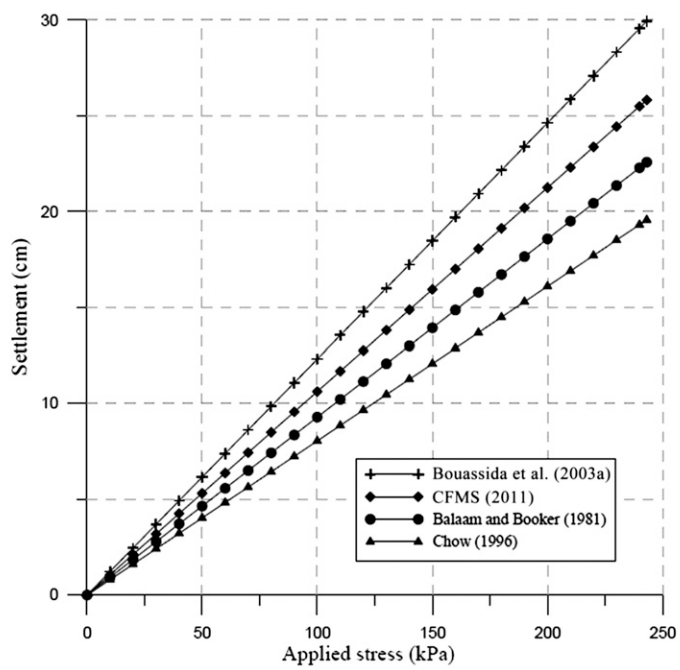

Fig. 4. Predicted settlement as a function of applied load for a tank on sand compaction columns

Furthermore, Fig. 5 shows that the optimized area ratio, as predicted by the *Columns 1.0* software (Bouassida and Hazzar 2012), increases from 14.6 to 36.3% when the allowable settlement of the CRF decreases from 16 to 9 cm. Such a case study illustrates that the main purpose of reinforcement by sand columns is to reduce settlement rather than to increase bearing capacity.

Embankment on Soft Clay Reinforced by Floating Columns

A road embankment 2 m thick and 16 m wide at its base was planned to be constructed on a soft clay layer that extends to 15 m beneath the ground surface (Fig. 6). The soil profile is composed of two layers.

The first layer was either cemented stabilized clay (CSC) or compacted sand fill (CSF) at a depth ranging from 1 to 5 m. The second layer was natural soft clay that extends to 15 m beneath the ground surface. To simulate different levels of traffic live loading, an additional surcharge q, varying from 10 to 50 kPa, was applied to the crest of the embankment. The weight of the embankment and the live loading were used in all settlement predictions.

The stability of the embankment was analyzed numerically to assess the performance of two proposed soil-improvement techniques suggested by Saadeldin et al. (2011). The first technique would have involved treatment of the soft clay by continuous cement stabilization (CSC) of the upper layer, and the second technique was the substitution of the upper portion of the soft clay by compacted sand fill (CSF).

Instead of substituting the foundation layer beneath the embankment to a depth of 5 m by CSC or CSF, as assumed by Saadeldin et al. (2011), reinforcement of the soft clay by sand columns and columns formed by cement deep-soil mixing also has been studied (Bouassida et al. 2012). The design procedure, incorporated in the *Columns 1.0* software (Bouassida and Hazzar 2012), consisted of estimating the optimized IAR that complies with the allowable bearing capacity and settlement criteria for these latter two foundation options.

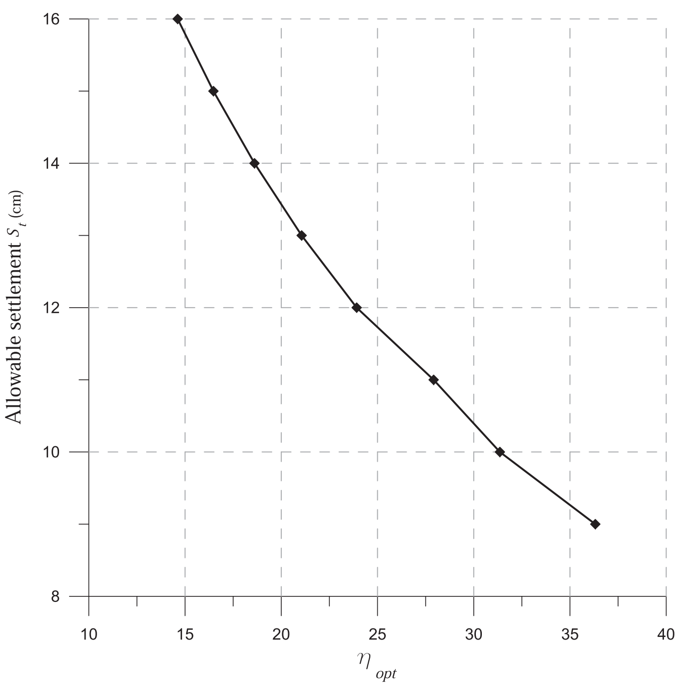

Fig. 5. Evolution of optimized IAR versus allowable settlement of reinforced clay by end-bearing sand columns

The optimized IAR depends on the maximum allowable settlement, on the applied surcharge pressure, and on the depth of reinforcement. It was assumed that the allowable settlement equals 10 cm for the long-term embankment behavior. Once the optimized IAR is determined, the software computes, for different diameters, the number of columns to be installed and, consequently, the volume of reinforcing material to be added. It is then easy to compare the volume of material substituted in the scheme adopted by Saadeldin et al. (2011), i.e., $\eta = 100\%$, with that predicted by the software. Tables 1 and 2 present the optimized IAR, the percentage of reduced material, for compacted sand and CSC columns, respectively. It can be concluded that the reinforcement by columns is much more economical than the technique involving full substitution of weak soil to 5 m beneath the soil surface, as suggested by Saadeldin et al. (2011). Indeed, even if the lengths of the columns exceeds 5 m, the optimized IAR would definitely be much less than that for total substitution ($\eta = 100\%$).

This second example illustrates again the efficiency of the software in estimating a truly cost-effective CRF.

Fig. 7 represents a linear trend of the increase in surcharge embankment load as a function of the optimized IAR, as predicted by the *Columns 1.0* software (Bouassida and Hazzar 2012) when half the thickness of the soft clay layer is reinforced by DMM floating columns 7.5 m in length. The optimized IAR is predicted such that the settlement of the reinforced soil layer is equal to 10 cm. Hence the linear variation from Fig. 7 is expressed by

$$q\,(\text{kPa}) = 40 + 0.9231\left[\eta_{\text{opt}}(\%) - 34.01\right] \tag{18}$$

Eq. (18) indicates a direct relationship between the exerted load applied by the embankment and the optimized IAR.

Trial Embankment in Saga, Japan

An embankment 6.5 m in height was constructed on the soil profile illustrated in Fig. 8, which shows floating DMM columns installed to a depth of 8.5 m (Chai and Carter 2011). This column reinforcement corresponds to a value of IAR of 30%, a value selected based on

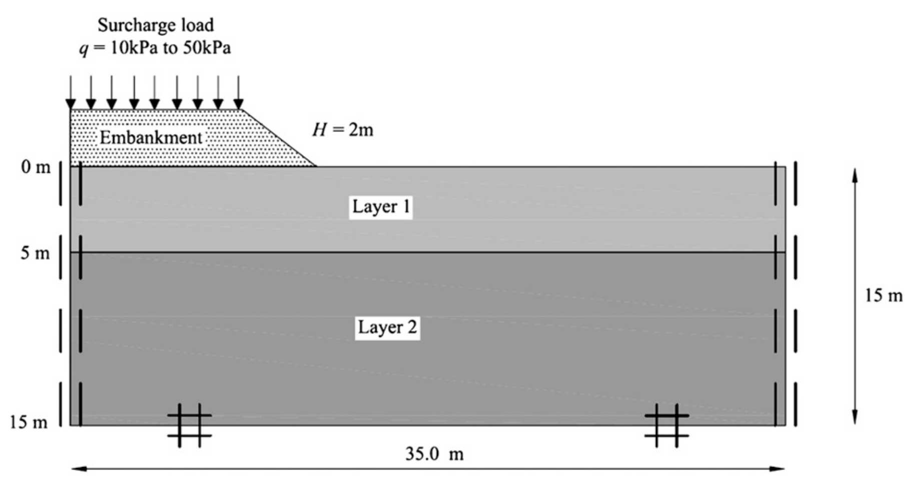

Fig. 6. Numerical model studied by Saadeldin et al. (2011), reprinted with permission from R. Saadeldin

Table 1. Optimized IAR for Sand Columns and Percentage Savings of Substituted Sand Fill

Surcharge (kPa)	Column length (m)	Optimized IAR (%)	Savings of substitution material (%)
10	7	32	55
20	7.5	17	75
30	7.5	31	54
40	7.5	44.5	33
50	7.5	58	12.5

Table 2. Optimized IAR for Deep-Mixed Soil-Cement Columns and Percentage Savings of Substituted CSC

Surcharge (kPa)	Column length (m)	Optimized IAR (%)	Savings of substitution material (%)
10	7.5	47	29
20	7.5	56	15.5
30	7.5	60	10
40	8	31	53
50	8	31	50

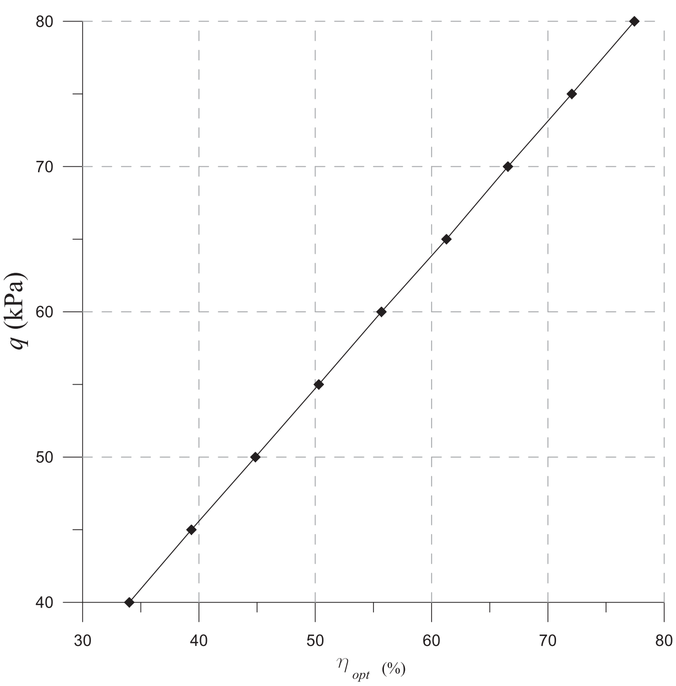

Fig. 7. Evolution of load embankment versus optimized IAR of soft clay reinforced by floating DMM columns of length 7.5 m

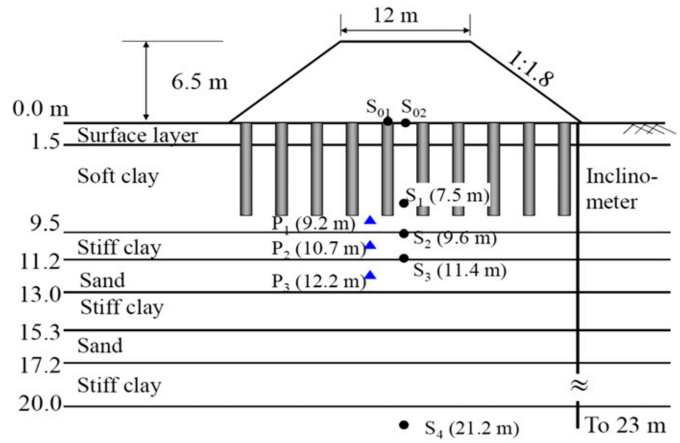

Fig. 8. Embankment trial project at Saga, Japan (Springer and *Geotechnical, Geological and Earthquake Engineering*, vol. 18, 2011, "Soil-Cement Columns," J. Chai and J. P. Carter, fig. 5.36, © Springer Science + Business Media B.V. 2011. With kind permission from Springer Science and Business Media.)

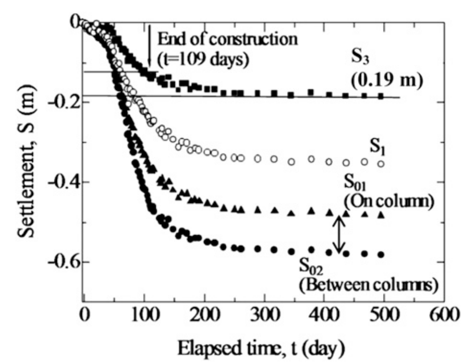

Fig. 9. Observed settlements for Saga case history (Springer and *Geotechnical, Geological and Earthquake Engineering*, vol. 18, 2011, "Soil-Cement Columns," J. Chai and J. P. Carter, fig. 5.37, © Springer Science + Business Media B.V. 2011. With kind permission from Springer Science and Business Media.)

experienced in other similar projects. The columns were installed within the soft Ariake clay layer, which is characterized by undrained shear strength of 12 kPa and a relatively high compression index C_c of 2. Instrumentation for this trial embankment consisted of settlement recorders placed (1) at the surface of the reinforced ground, i.e., points S_{01} and S_{02}, located, respectively, at the column head and on the original surface between adjacent columns and (2) in the reinforced soil mass at point S_1 and within the unreinforced soil layer at points S_2 and S_3. The evolution of the observed settlements is given in Fig. 9, on which the following remarks are based.

At the surface of the reinforced ground, significant differential settlement, about 10 cm, has been observed. This is essentially attributed to the big difference between the rigidities of the soft clay and column material. This behavior would not be expected if a thin mattress layer, at least approximately 30 cm thick, made up of well-drained material had been spread out over the ground surface to make the surface settlement of the reinforced ground as uniform as possible.

The long-term settlement of the unreinforced layers, about 19 cm in magnitude, is attained over a period of about 300 days. However, at the end of embankment construction, about 60% of this settlement (approximately 11 cm) had already occurred. The latter can be regarded as short-term settlement, corresponding to completion of the loading of the reinforced soil.

The Saga case history has been investigated by carrying out the complete design procedure incorporated in the *Columns 1.0* software (Bouassida and Hazzar 2012), comprised of verifications of the bearing capacity and settlement. Table 3 summarizes the geotechnical properties of the soil layers adopted by Chai and Carter (2011), from which the characteristics of the column material have been deduced, i.e., $E_c = 30{,}000$ kPa, $\nu_c = 0.3$, $s_u = 300$ kPa, $\varphi_c = 0$, and $\gamma_c = 17\,\mathrm{kN/m^3}$. Using these input data in the *Columns 1.0*

Table 3. Geotechnical Properties of Soil Layers at Saga Site

Layer number	Thickness (m)	Undrained shear strength (kPa)	Young's modulus (kPa)	Poisson's ratio	Friction angle (degrees)	Unit weight (kN/m^3)
1	1.5	15	2,000	0.33	0	20
2	4.5	10	1,500	0.45	0	13.5
3	3.5	20	3,000	0.4	0	14.3
4	1.7	25	4,000	0.35	0	18
5	1.8	0	15,000	0.33	35	18
6	2.29	45	10,000	0.35	0	18
7	1.9	0	17,000	0.3	38	18.5
8	2.8	60	13,500	0.33	0	19

software and assuming surface loading corresponding to an embankment having the dimensions given in Fig. 8 and a unit weight of the embankment fill of 18.2 kN/m^3, the following predictions were obtained:

1. Verification of bearing capacity. It was found that a minimum IAR of 10.22% would be required to comply with an allowable bearing capacity corresponding to a safety factor of 2.
2. Verification of settlement. Because floating columns were installed, verification of settlement was achieved by comparing the total settlement, composed of the sum of the settlements of the reinforced soil and the underlying unreinforced soil layer, with the allowable settlement. In addition, as observed in Fig. 9, most of the settlement of the reinforced soil occurs at the end of construction. Accordingly, the optimized IAR has been sought in the range of total allowable settlement between 20 and 32 cm. From the settlement predictions obtained by the *Columns 1.0* software, the following interpretation has been made.

It is also of interest to use *Columns 1.0* to predict how the overall settlement of the reinforced foundation system would vary as a function of IAR for the specified loading over the foundation, i.e., $6.5 \times 18.2\,\mathrm{kN/m^3} = 118.3$ kPa. Fig. 10 shows the variation in settlement as a function of IAR, as predicted by the method of Balaam and Booker (1981). Plotted in this figure are curves corresponding to the overall settlement of the foundation system, as well as that component of the overall settlement contributed by the layer of soil that is reinforced by columns. Obviously, the vertical separation of these two curves corresponds to the contribution to the overall settlement of the underlying unreinforced soil layer. Fig. 10 indicates that the larger the allowable settlement of the reinforced soil, the smaller will be the required value of IAR over the range of values of IAR $> 10.22\%$. The lower limit on IAR is required to satisfy the allowable-bearing-capacity criterion.

It would seem reasonable to consider that the average settlement within the reinforced soil mass at the end of embankment construction (as recorded at locations S_1) is approximately 32 cm. Further, it can be seen in Fig. 10 that a total allowable settlement of 30 cm corresponds to an optimized IAR of about 16%. Therefore, it can be concluded that there has been little benefit in constructing the columns with a value of IAR as large as 30%. The additional area (and volume) of column material corresponding to the higher value of IAR was unnecessary for the settlement criterion to be met. Perhaps the main advantage of using a value of IAR as large as 30% would appear to be as compensation for the nonuse of a spreading layer of drained material at the ground surface to control the differential settlements.

From the three cases histories analyzed and presented in this section, it can be concluded that the *Columns 1.0* software provides a simple and efficient tool for predicting the optimized design of CRFs, both for end-bearing and floating columns.

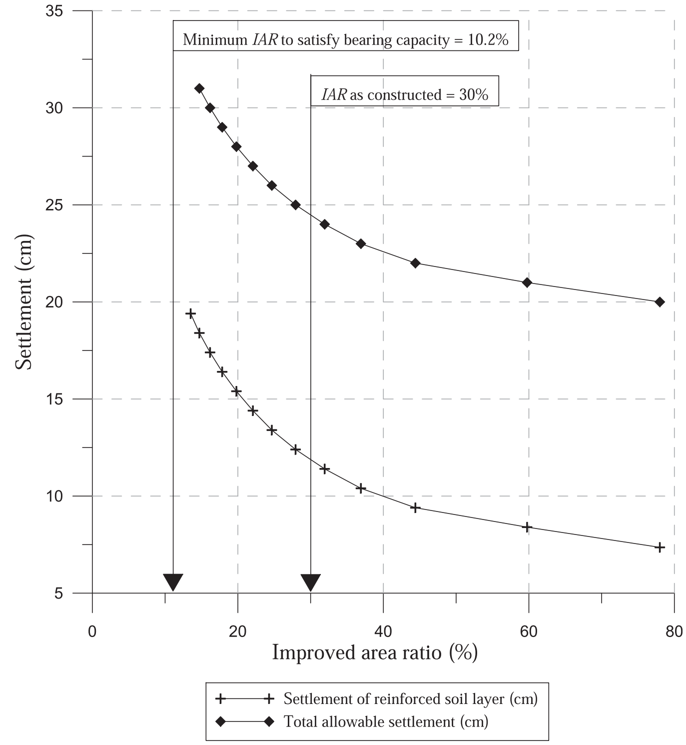

Fig. 10. Settlements predicted by *Columns 1.0* for Saga case history as a function of optimized IAR

Conclusions

A method has been suggested for carrying out the design of CRFs in soft ground. This comprehensive methodology successively considers the required bearing capacity and settlement criteria in the design procedure, in contrast with previous methods of design that focus only on a unique criterion of design, i.e., settlement or bearing capacity alone. The latter methods are usually dedicated to some specific ground-improvement technique. The main advantages of the suggested design method are its validity for all techniques of column installation and its applicability to both end-bearing and floating columns.

The IAR, which normally controls the cost of soil treatment, was targeted as a quantity to be optimized over a given range, satisfying the performance requirements of the foundation in terms of allowable bearing capacity and settlement. For practical purposes, the methodology has been implemented in the *Columns 1.0* software, which provides a viable tool for optimized and interactive design, being applicable to a variety of geotechnical structures.

Three different cases of study have been analyzed by the software. Predicted results demonstrate the efficiency of the proposed methodology in term of cost-effectiveness, specifically avoiding overdesigned solutions.

Appendix. Reinforcement of a Purely Cohesive Soil with Cohesive Frictional Column Material

Expressions for the ultimate vertical stresses acting on the column material and host soil appearing in Eq. (2), i.e., $Q^{-}_{\mathrm{ult}}/A = \sigma_{\mathrm{ult},rs} = (1-\eta)\sigma_{\mathrm{ult},s} + \eta\sigma_{\mathrm{ult},c}$, are given effectively as follows (after

Bouassida et al. 1995): For $1 \leq K_{pc} \leq 2$, columns are located in an arbitrary pattern under the loaded foundation

$$\left(\frac{Q_{\text{ult}}^{-}}{A}\right)_{K_{pc}\leq 2} = 4C_s(1-\eta) + 2\eta\left(C_s K_{pc} + C_c\sqrt{K_{pc}}\right)$$

For $2 \leq K_{pc} \leq 1 + (b/a)^2$, columns of identical radius a are not necessarily located in a regular pattern under the loaded foundation. b is the minimum distance between column axes.

$$\left(\frac{Q_{\text{ult}}^{-}}{A}\right) = \left(\frac{Q_{\text{ult}}^{-}}{A}\right)_{K_{pc}\leq 2} + \eta C_s g(\varphi_c)$$

where

$$g(\varphi_c) = \left(K_{pc}-1\right)\ln\left(K_{pc}-1\right) - \left(K_{pc}-2\right)$$

The symbol $K_{pc} = tg^2(\pi/4 - \varphi_c/2)$ is used to denote the coefficient of passive earth pressure of the column material.

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