

Optimized Design of Foundations on Soft Soil Reinforced by Floating Granular Columns

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Abstract: This paper studies the design of foundations built on thick compressible soft soil layers that are reinforced by floating columns. Based on a recent methodology, the suggested design combines the bearing capacity and settlement verifications to provide an optimized improvement area ratio (IAR). Then, an optimized length for the floating columns is obtained by introducing the admissible long-term settlement of the unreinforced compressible sublayers and assuming that the total short-term settlement vanishes at the end of project construction. This paper focuses on the variation in the consolidation settlement of the unreinforced compressible sublayer versus the length of the floating columns. The discussion of this design methodology highlights the feasibility of a potential reinforcement solution when producing a cost-effective design, which assures an optimized IAR within the reinforcement with end-bearing columns is not required to control the admissible long-term settlement. Instead, the suggested design method enables the determination of the optimized length of the floating columns, which satisfies the admissible residual settlement and consolidation time. The comparison between the proposed results and numerical predictions by Plaxis 2D shows good agreement, which confirms the feasibility of an optimized length for floating columns and avoids the systematic adoption of end-bearing reinforcement in columns. **DOI: 10.1061/IJGNAI.GMENG-9259**. © *2024 American Society of Civil Engineers*.

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Introduction

The reinforcement of very thick soft deposits is challenging for several reasons, in particular, for heavy-loaded structures (e.g., high embankments) and constructions that are sensitive to differential settlement, such as storage facilities. In addition, the determination of the reinforcement depth requires care whenever a very deep stratum layer exists. Before the reinforcement by rigid inclusions or pile foundations was analyzed, a floating column-reinforced foundation was the most suitable approach for these projects. Bergado et al. (1996) reported that these issues were addressed with floating stone columns. In addition, the reinforcement by floating cement– soil columns in soft clays, which focused on the consolidation settlement, has been studied (Chai and Carter 2011). Published papers over these last few decades about reinforcement using floating columns have not addressed, in the various case studies, a full design including the verifications of bearing capacity, with a subsequent

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emphasis on admissible settlements and the time of consolidation. These contributions have not raised the questions of the area ratio or floating column length optimizations. Most contributing papers focused on experimental and numerical investigations on the spacing between columns and the effect of the diameter and length on the bearing capacity, foundation settlement, or both when built on reinforced soil. Fattah et al. (2017) carried out an experimental program to estimate the bearing capacity of a floating stone column group that was installed in clays with different undrained shear strengths and different diameters and lengths, which were installed with the casing bored method. Their results did not discuss the settlement and consolidation time of the unreinforced subsoil layer. From the numerical contributions, Abuelgasim et al. (2021) reported the observed deformation of floating columns, which did not justify their length.

The feasibility of reinforcement that uses floating columns relies on three verifications: (1) the stability against punching of the unreinforced soft sublayer; (2) the allowable settlements of the reinforced and unreinforced soil; and (3) the evolution of long-term settlement in the unreinforced sublayers. Bouassida and Carter (2014) highlighted a combination of the bearing capacity and settlement verifications in a general framework, by which all types of reinforcement by columns could be treated, that would lead to an optimized improvement area ratio (IAR), which ensured a costeffective design for the column-reinforced foundation. After studying several case histories, using the methodology implemented in Columns 1.01 (Bouassida and Hazzar 2012) accordingly, a costeffective optimized design is targeted. Tabchouche et al. (2017) studied the behavior of compressible soil that was reinforced by a group of end-bearing columns. Settlement predictions with FLAC3D supported the validation of various implemented numerical models using field data. Bouassida et al. (2009) analyzed the ultimate bearing capacity of a foundation that rested on soil reinforced with a group of floating columns. From this analysis, the maximum length of the columns was identified. This was the first

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requirement, which made the determination of the lower bound of the ultimate bearing capacity possible. Lorenzo and Bergado (2003) introduced a solution to estimate the degree of consolidation for floating soil-cement column improvement. The governing equation describes the interdependent consolidation of the soil-cement column with its surrounding clay, which uses the unit cell model (UCM). This model ignores the vertical consolidation that is induced in natural soft soil. Miao et al. (2008) proposed a method that considers the effect of the higher stiffness of the column without considering the drainage effect of the columns.

The Japanese Institute of Civil Engineering (JICE) suggested a design method when calculating the settlement of a foundation that rested on soft soil improved with a floating soil–cement columm (ASCRDO 2009). When the IAR is equal to or more than 30%, from the practice the JICE method considers that the unreinforced sublayer, in terms of settlement, is responsible. When the IAR is <30%, the unreinforced sublayer and the third thickness of the columns' reinforced upper layer prevail in the settlement estimation. The effectiveness of the JICE method has been demonstrated by comparing the calculated settlement values with the recorded data for three case histories in Fukuoka, Japan. As an extension of the JICE method, Chai et al. (2010) proposed the α – β method to calculate the final consolidation settlement of the soft deposit that was improved with floating columns.

Throughout the synthesis of the existing contributions of the previous methods the design, as commented at the beginning of this introduction, it is clear that the optimization of the floating column length requires deeper analysis, which includes the evolution of the residual settlement of the unreinforced soil. This paper aims to demonstrate the feasibility of reinforcement with columns shorter than the rigid stratum depth in a very thick soft deposit. This reinforcement, termed *floating columns*, is viable because the long-term settlement of the compressible, unreinforced soft soil will not affect the serviceability of the structure that is built on the reinforce a very thick soft clay layer? Bouassida and Hazzar (2015) implemented this methodology for design. However, they did not focus on the required consolidation time for the evolution of the residual settlement of the unreinforced soil.



Fig. 1. Geotechnical profile of the case study.

This paper will determine an optimized IAR. Then, the consolidation time will be discussed, which leads to optimizing the floating column length based on the evolution of the residual settlement. Finally, the obtained long-term settlement for the compressible soft soil is compared with those predicted using the JICE method. An assessment of the obtained results with the proposed design method is discussed based on the numerical investigation performed with Plaxis 2D, which uses the UCM of soft soil reinforced by a floating column.

Proposed Method of Design for Foundations on Reinforced Soils by Floating Columns

Case Study Investigation

The suggested design method in this paper focuses on the settlement verifications for a foundation built on soil reinforced by floating columns for the short and long-term conditions. The implementation of this design method considers the case study investigated by Bouassida and Carter (2014), which was first performed for the reinforcement design for end-bearing columns, and the soil profile is shown in Fig. 1. This soil profile is composed of two saturated soft clay layers, which are 10 m thick and overlay an impervious rigid stratum at a depth of 20 m from the soil surface.

The foundation consists of a square rigid raft of width B = 15 m and an applied load at the ground surface, equivalent to a uniform vertical stress equals 80 kPa.

Tables 1 and 2 summarize the geotechnical parameters of the two soft clay layers. Based on Mezni and Bouassida (2019), the oedometer parameters of the two soft clay layers, which were assumed to be normally consolidated, are: (1) compression index ($C_c = 0.35$); (2) initial void ratio ($e_0 = 0.9$); and (3) coefficient of vertical consolidation ($c_v = 2.10^{-8} \text{ m}^2/\text{s}$).

First, the feasibility of the case studied (Fig. 1) required the verification of the bearing capacity of the square rigid raft with a zero embedment depth. Using the bearing capacity from Terzaghi's equation (cohesion and friction angle parameters) (Das 2021), it is simple to confirm for a shallow foundation that the admissible bearing capacity in the short-term case for the raft foundation with zero embedment equals 45 kPa. Therefore, it is less than the working load (q = 80 kPa). Reinforcement of the upper soft clay layer with floating columns was required.

Table 3 lists the linear elastic and strength parameters of the column material adopted for the design of the squared raft foundation on soil reinforced by floating stone columns.

Table 1. Geotechnical parameters of clay 1 layer

Thickness (m)	$\gamma (kN/m^3)$	C (kPa)	φ (°)	E (kPa)	v
10	17	20	0	3,000	0.40

 Table 2. Geotechnical parameters of clay 2 layer

Thickness (m)	$\gamma (kN/m^3)$	C (kPa)	φ (°)	E (kPa)	v
10	18	25	0	3,500	0.35

Table 3. Geotechnical parameters of column material

$\gamma_{\rm col}~({\rm kN/m^3})$	$C_{\rm col}~({\rm kPa})$	$\varphi_{\rm col}$ (°)	$E_{\rm col}~({\rm kPa})$	$V_{\rm col}$
19	5	40	30,000	0.25

Determination of the Optimized IAR

For the previously studied case, the methodology of design detailed by Bouassida and Carter (2014) and implemented in Columns 1.01 determines the minimum IAR that complies with the bearing capacity verification first. Then, it addresses the settlement verification in the short-term condition. This verification postulates that the total settlement should comply with the agreed admissible value. This second verification leads to the identification of the optimized area ratio (η_{opt}).

If a total allowable short-term settlement ($\delta_{tot,adm} = 7.5$ cm) is considered, which considers a normalized settlement <1% ($\delta_{tot,adm}/B = 0.005$), from the analytical results obtained with Columns 1.01, the minimum length of the floating columns is $H_c =$ 8 m. The corresponding optimized area ratio (η_{opt}) is 34.64%. Of note, the latter is the highest area ratio that could be implemented in practice for the installation of stone columns (Bouassida 2016). Therefore, the first design step ends with determining the optimized area ratio (η_{opt}), which corresponds to the minimum value of the length of the floating columns (H_{c0}).

Referring to Bouassida and Hazzar (2015), the first step in the design method based on the linear elastic method suggested by Bouassida et al. (2003) predicts the short-term settlement of the reinforced upper layer. It is assumed that the short-term settlement of the unreinforced sublayer(s) could be neglected compared with the primary consolidation settlement. Subsequently, the optimized area ratio (η_{opt}) is adopted using Columns 1.01, based on the admissible value of the total short-term settlement (Bouassida and Hazzar 2012).

Then, the optimized length of the floating columns (H_c) and the consolidation time are determined based on the admissible long-term settlement of the unreinforced sublayer(s) of thickness H_u (Fig. 2).

The predictions of the settlement, such as the short and longterm components, coincide with the geometrical axis of the squared raft foundation.

Because the reinforcement cost is proportional to the volume of the incorporated reinforcing material (i.e., corresponds to the IAR and the length of the stone columns), this novel method could adopt an optimized design for floating column-reinforced foundations.

Optimized Length of Floating Columns

The identification of the optimized length for floating stone columns is calculated using a parametric study where the length of the floating columns are $H_c = 8$, 10, 12, 14, 16, and 18 m, as shown in Fig. 2, and the total admissible short-term settlement equals 7.5 cm. In addition, a 0.5 m thick drainage layer at the top of the reinforced ground enables water evacuation that results from the consolidation of the reinforced soil layer.

Fig. 3 shows the variation in the short-term settlement of the unreinforced sublayer (H_u) using Columns 1.01 versus the length of the floating columns. Of note, $H_c = 16$ m corresponds to the negligible short-term settlement in the unreinforced soft clay sublayer (i.e., < 1.5 cm). This result explains why end-bearing columns (embedded in the rigid stratum) are not a potential in the design in the short-term condition.

Fig. 3 shows that the greater the length of the floating columns, the more the assumption of the zero horizontal displacements is valid owing to the slimness of the unreinforced sublayer (H_u) , which decreases as the column length (H_c) increases. Therefore, the settlement predicted by Columns 1.01 is similar to the settlement predicted when zero horizontal displacement was assumed (the oedometer modulus).

Fig. 4 shows the variation in the optimized area ratio (η_{opt}) that was obtained by Columns 1.01 versus the column lengths when the admissible total short-term settlement was $\delta_{tot,adm} = 7.5$ cm. From this figure, for column lengths from $H_c = 8$ to 12 m, the optimized area ratio decreases from 35% to 16%. Therefore, a shorter column leads to a higher optimized area ratio for a prescribed allowable short-term settlement. In addition, the IAR depends on the column length; the IAR decreases up to 9.2% when the length of the floating columns is 18 m.

After calculating the volume of the incorporated reinforcing material and the total cost of the improvement, it is possible to check that it is almost constant for the columns' length $H_c = 14$, 16, and 18 m $H_c\eta_{opt} = 401.625$, 383.400, and 373.005 m³, respectively (Fig. 5). The relative difference between the volumes of reinforcing material that corresponds to 14 and 18 m column lengths is negligible at 7.67%.









goes consolidation settlement (Fig. 1). The optimized length of floating columns (H_c^{opt}) contributes to the admissible value of the residual long-term settlement of the unreinforced compressible layer.

Estimation of the Long-Term Settlement

The long-term settlement of the unreinforced clayey sublayer (*i*) each of thickness H_{ui} and assumed normally consolidated can be estimated by applying Terzaghi's 1D method (Bouassida and Hazzar 2008)

$$s_i = H_{ui} \frac{C_c}{1 + e_0} \log\left(1 + \frac{\Delta\sigma}{\sigma'_{\nu 0}}\right) \tag{1a}$$



Fig. 5. Volume of the incorporated material versus the length of floating columns (H_c) .

where $s_i = \text{consolidation}$ settlement of the compressible sublayer of thickness H_{ui} ; $C_c = \text{compression}$ index; $e_0 = \text{initial}$ void ratio; $\Delta \sigma = \text{excess}$ of vertical stress induced by the square raft; and $\sigma'_{v0} = \text{effective}$ overburden stress. Both are calculated in the middle of the thickness of the compressible sublayer *i*. In this paper, the small thickness of the unreinforced sublayers equals 2 m is suitable to estimate the excess vertical stress ($\Delta \sigma$), which decreases rapidly with depth. Using the following equation, this excess vertical stress is calculated in each sublayer (*i*) at the axis of the square raft foundation (B = L = 15 m) subjected to the uniform load q = 80 kPa (Fig. 1), at a given depth using the chart proposed by the US Navy (1971):

$$\Delta \sigma = I\left(\frac{B}{z}, \frac{L}{z}\right) \times q \tag{1b}$$

The 1D consolidation theory assumes that the horizontal displacement component is negligible compared with the settlement value. This assumption applies when the thickness of the unreinforced soil layer is ¹/₃ (or less) of the foundation dimensions. The calculated long-term settlement in this paper considers unreinforced soil sublayers 2 m thick, which is very small compared with the width of the raft square foundation equals 15 m. To highlight the effect of horizontal displacement on the calculation of long-term settlement, the correction proposed by Skempton and Bjerrum (1957) could be adopted. In the Appendix, a detailed presentation of this correction method that was applied to the studied case with the related data shows that the effect of the horizontal displacement approaches zero, and, therefore, the adopted Terzaghi's 1D settlement calculation of the unreinforced soil is valid.

The adopted method of settlement estimation for the unreinforced soil does not consider the effect of column installation by improving the upper soft layer (Frikha et al. 2013; Ellouze et al. 2017).

The second step in the design is based on verifying the allowable long-term settlement in the unreinforced sublayer. The evolution of the long-term settlement versus time could offer another feature of the behavior of the foundation on the layer that is reinforced by floating columns. The column length optimization depends on the residual settlement of the unreinforced clayey sublayer as determined from

$$\delta_{\rm res} = \delta_{\rm LT} - \delta_{\rm ST} \tag{2}$$

where $\delta_{LT} =$ long-term settlement determined from Eq. (1); and $\delta_{ST} =$ short-term settlement determined by the linear elastic method embodied in Columns 1.01. Fig. 6 shows the evolution of the short and long-term and residual settlement components of the unreinforced clay sublayer versus the length of the floating columns. As shown in Fig. 6, the optimized length of the floating columns equals 16 m. Based on the proposed design, the corresponding residual settlements were <4 cm, which is an admissible value compared with the width of the rigid raft of 15 m.

This result indicates that the long-term stability of the raft foundation that uses floating stone columns is viable. Therefore, reinforcement that uses end-bearing stone columns is not required to comply with the rafted foundation's long-term stability.

Validation of the Proposed Method

Estimation of the Long-Term Settlement Using the JICE Method

The JICE (1999) proposed a design method used in Japan to predict the consolidation settlement of soft soil treated with floating cement columns (ASCRDO 2009).



The estimation of the consolidation settlement using the JICE method depends on the IAR and the depth improvement ratio (β). This ratio (β) is a function of the length of the floating column (H_c) and the thickness of the soft deposit layers (H). When the IAR is <30% (η < 0.3), the JICE method assumes that the total settlement equals the sum of the settlements of the unreinforced sublayer and the settlement of $\frac{1}{3}$ the thickness of the reinforced upper layer. Then, when the IAR is equal to or >30% ($\eta \ge 0.3$), the unreinforced sublayer. Comparisons between the calculated settlements that used the JICE method and recorded data from three case histories in Fukuoka, Japan, proved the effectiveness of this method.

By implementing the JICE method, explained in the previous section, the long-term settlement could be calculated with Terzaghi's 1D method using Eq. (1) after the excess vertical stress has been calculated within the unreinforced sublayer of thickness H_{uJICE} from the following equations for $\eta < 30\%$ and $\eta \ge 30\%$, respectively:

$$H_{u,JICE} = (H_u + H_c/3) \tag{3a}$$

$$H_{uJICE} = (H - H_c) \tag{3b}$$

Fig. 7 shows the settlement that was estimated by the JICE method, and the long-term settlement predicted from Eq. (1). As shown in Fig. 7, the long-term settlement that was estimated by the JICE method significantly overestimated the one predicted in this paper when the length of the floating columns varied between 10 and 16 m. When the column length was $H_c = 8$ m, the optimized area ratio exceeded 30% ($\eta_{opt} = 34.59\%$). Therefore, only the settlement contribution from the unimproved sublayer was considered. The JICE and proposed methods in this paper provided equal long-term settlement. The discrepancies shown in Fig. 7 were attributed to the assumed increase in the thickness of unreinforced soft soil that was proposed by the JICE method when the column length was from 10 to 16 m for $\eta < 0.3$ (Table 3). This led to the overestimation of the long-term settlement by the JICE method.



Fig. 7. Comparison between settlement predictions by the proposed and JICE methods.

Evolution of the Time of Consolidation

Based on Terzaghi's 1D consolidation theory, the consolidation time (t_c) is calculated from

$$t_c = T_v H_u^2 / c_v \tag{4}$$

where H_u = thickness of the unreinforced sublayer(s), which was assumed to equal the length of the drainage path due to the presence of the impervious stratum (Fig. 1). However, the evacuated water from the top side of the unreinforced sublayer occurred through the toe of the floating columns. Therefore, at depth $z = H_c$, water drainage through the reduced area was $\eta_{opt}B^2$ (the area of the tip of the columns), which was significantly less than the total loaded area. Therefore, the consolidation time of the unreinforced sublayer(s) was greater than that of a full drainage layer at depth z = H_c . Therefore, from Eq. (4), the calculated consolidation time was underestimated. In Eq. (4), the time factor (T_v) was determined from the consolidation degree as a solution to Terzaghi's consolidation theory. The following equation defines the relationship between the degree of consolidation (U_v) and the time factor

$$U_{\nu} = \left(\frac{T_{\nu}^{3}}{T_{\nu}^{3} + 0.5}\right)^{\frac{1}{6}}$$
(5)

Fig. 8 shows the variation in the long-term settlement of the underlying unreinforced layer versus time for various lengths of floating columns. The consolidation settlement decreased by increasing the column length. Installing longer columns led to an accelerated degree of consolidation (U_v) . Therefore, the long-term settlement decreased. Therefore, for $H_c = 8$ m, the consolidation rate after 150 years for the floating column was approximately 80%. By increasing the length of the floating columns, the predictions indicated a 100% average degree of consolidation after 10 years for $H_c = 18$ m.

The prediction shown in Fig. 8 confirmed the efficiency of the reinforcement by floating stone columns based on the variation in consolidation settlement with time.

In addition, Fig. 8 shows the adoption of different lengths of floating columns, which depend on the admissible long-term settlement. When the long-term settlement was 3 cm, two different lengths of columns could be adopted: $H_c = 14$ m after 7 years consolidation time or $H_c = 16$ m for a longer period (17 years consolidation time).



Fig. 8. Settlement versus time for different length of floating stone columns.

This prediction might allow flexibility in practice when the consolidation time is required first rather than the magnitude of settlement. In addition, Fig. 8 shows that reinforcement by floating stone columns <14 m long was unacceptable in practice due to the nonadmissible long-term settlement of the unreinforced soft soil layer.

Therefore, reinforcing soft clay with floating stone columns 16 m long revealed a safer scenario. After raft construction, which was assumed to be 2 years, the residual consolidation settlement did not exceed 1.50 cm, which is an effective admissible limit for the stability of the rigid raft 15 m wide.

The strength of the proposed methodology relies on its simplicity to optimize the length of the floating columns, and end-bearing columns are not required as a solution.

Assessment of the Proposed Method: Numerical Analysis

The section assesses the suggested optimized length of the floating columns, which considers the admissible residual settlement and consolidation time of the unreinforced soft soil detailed in the previous section. This assessment relies on the numerical predictions that were performed with Plaxis 2D (version 8).

Numerical Model

The axisymmetric UCM reproduced the 1D consolidation test well under the oedometer conditions of the case study shown in Fig. 1.

Geometrically, the UCM was composed of a single stone column within an infinite grid of stone columns, which was surrounded by a cylinder that enclosed its tributary soil (Elshazly et al. 2008; Ng and Tan 2014). The equivalent diameter of the UCM (d_e) represents the periodic influence zone of the installed column in a regular pattern with an axis-to-axis spacing between the columns (S_c).

For the case studied, a group of 0.80 m diameter stone columns that were installed in a triangular pattern was adopted. The equivalent diameter of the domain of influence of each column is

Table 4. Axis-to-axis spacing between columns of 0.8 m diameter versus optimized IAR and column length

H_c (m)	η_{opt} (%)	S_c (m)	N_c
8	34.59	1.29	154
10	21.89	1.62	97
12	16.05	1.90	71
14	12.75	2.13	57
16	10.65	2.33	47
18	9.21	2.51	41

(Balaam and Booker 1981)

$$d_e = 1.05S_c \tag{6}$$

Using Columns 1.01 for the identified optimized area ratio (η_{opt}) and a given pattern of column installation, the axis-to-axis spacing between the columns (S_c) and the number of columns (N_c) were determined. Table 4 summarizes the values of the axis-to-axis spacing as a function of the optimized IAR and column length using Columns 1.01.

The Mohr–Coulomb constitutive law was adopted for the soft soil and the stone column material with the geotechnical parameters given in Tables 1–3, respectively. Due to symmetry, the adopted model was reduced to half of the influence zone. Fig. 9 shows a vertical cross section of the simulated UCM with the generated mesh composed of 15-noded triangular elements and the boundary conditions that complied with the assumed oedometer condition (i.e.,



Fig. 9. Typical cross section of the UCM with a floating column $H_c = 8$ m.

zero horizontal displacements at the lateral border of the UCM). The phreatic level was set at the surface of the UCM. Fig. 9 shows that the radii of the granular column and the UCM were $r_c = d_c/2$ and $r_e = d_e/2$, respectively.

Numerical Results

The numerical analysis focused on the consolidation settlement in the unreinforced soil by increasing the length of the floating columns from $H_c = 8$ to $H_c = 18$ m (every 2 m).

First, this paper investigated the influence of the length of the floating columns on the reinforcement's performance for settlement reduction. At this stage, an undrained analysis was performed for the reinforced soil layer.

Second, this paper focused on the prediction of the consolidation time for the unreinforced sublayer when the length of the floating stone columns increased, therefore validating the proposed method.

Settlement Response of the Floating Column-Reinforced Soil

Fig. 10 shows the settlement predictions for the unreinforced sublayer at the tip's axis of the floating column, which corresponded to the optimized IARs that were determined with Columns 1.01 when the UCM diameter varied from 1.355 to 2.635 m by the optimized IAR and the adopted diameter of the stone columns.

These settlement predictions by Plaxis 2D and Columns 1.01 were in good agreement for lengthy floating columns (i.e., $H_c \ge$ 14 m). However, the predicted settlement was underestimated when the length of the floating columns was $H_c < 14$ m.

The difference between the two settlement predictions could be explained as follows. In the upper side of the UCM, the lateral confinement that was provided by the expansion of the column in the surrounding soft clay prevailed, which resulted from the zero horizontal displacement condition. This led to lower settlement compared with the unreinforced sublayer of the UCM. The predictions by Columns 1.01 correspond to a three-dimensional linear



Fig. 10. Comparison of short-term settlements predicted by Columns 1.01 and PLAXIS 2D.



Fig. 11. Comparison of consolidation settlement by the proposed method with PLAXIS 2D predictions.

elastic model (Bouassida et al. 2003), under the condition of possible horizontal displacement surrounding the reinforced soil beneath the loaded area that results in stiffer behavior of the overall improved ground and leads to the undue settlement.

Consolidation Settlement

A consolidation analysis was conducted to analyze the dissipation of excess pore pressure in the saturated clayey sublayer as a function of time. The long-term settlement beneath the floating column toe was predicted by adopting the consolidation, water drainage, and excess pore pressure dissipation: the right and left vertical boundaries (which represented the line of symmetry) were closed (impervious). In addition, the closed boundary was applied at the bottom of the UCM (the presence of an impervious layer). In turn, the upper boundary was open.

To demonstrate the validity of the new method, Fig. 11 shows the long-term settlement after 150 years for different H_c values, which used finite-element method (FEM) analysis. From Fig. 11, the increase in the floating column length reduced the settlement and, therefore, resulted in an accelerated consolidation rate due to the fast migration of water from the soft clay unreinforced soil into the floating column. However, when the floating column was shorter, the consolidation took time, and the water migrated slowly because of the thickness of the unimproved layer (H_u).

The two curves in Fig. 11 show good agreement between the predictions of the analytical method proposed in this paper and the numerical predictions with Plaxis 2D. The difference in the results is negligible and lies in the assumption of the consolidation theory that was implemented in the finite element package used in Plaxis, which endorses Biot's theory for coupled consolidation (Plaxis, Version 8). However, the proposed method adopted Terzaghi's 1D consolidation theory.

Conclusion

This paper addressed optimizing the length of floating stone columns as a reinforcement in a soft clay layer. A bibliographic review highlighted the absence of contributions where a comprehensive design could justify a meaningful floating column length as a competitive solution instead of the classical end-bearing column solution. Based on Terzaghi's 1D consolidation settlement, this paper suggested a design method to determine the optimized length of a floating stone column beneath the column toe.

The new method highlighted the feasibility of an alternative solution of in situ reinforcement in a cost-effective design, which assured an optimized IAR and floating column length that complied with an admissible long-term settlement associated with an acceptable consolidation time.

The results from the suggested analytical method were compared with those obtained using a two-dimensional FEM, which was performed with the UCM that was composed of a single floating stone column that was surrounded by a cylinder that enclosed its tributary soil. This comparison showed a good agreement between the analytical and numerical results. In addition, the main merit of the suggested analytical method demonstrated that reinforcement by end-bearing columns is not required to control the admissible long-term settlement in the unreinforced compressible layer. From the investigated case study, the reinforcement of a 20 m thick soft clay layer was safe by installing stone columns 16 m long with an admissible residual long-term settlement of 1.5 cm under a rigid square raft 15 m wide.

Appendix. Skempton–Bjerrum Method (1957)

The 1D deformation with the oedometer method does not always correspond to reality. Skempton and Bjerrum (1957) proposed improving this method for long-term settlement calculations. The inherent assumption of zero horizontal displacement does not prevail, for instance, when the width of the loaded foundation is shorter than the thickness of the compressible layer.

The Skempton–Bjerrum method contributed to the settlement analysis by highlighting that an element of soil underneath a foundation undergoes lateral deformation because of the applied loading. In addition, the induced pore water pressure is generally less than the increment in the vertical stress on the element because it depends on the pore pressure coefficient (A) (Skempton 1954).

Because the consolidation of clay results from the dissipation of the excess pore pressure, Skempton and Bjerrum (1957) proposed that a correction factor, the settlement coefficient (μ), which was introduced in the following equation, should apply to the settlement calculated based on the oedometer test:

$$s_{\rm cor} = \mu s_i \tag{7}$$

where $\mu =$ function of the geometry of the studied problem (*z*/*B*). The thickness of the sublayer is *z*, the width of the footing is *B*, and the value of coefficient *A* is introduced in

$$\mu = A + \alpha (1 - A) \tag{8}$$

In Eq. (7), the corrected settlement using the Skempton–Bjerrum method is s_{cor} and the settlement calculated from Eq. (1) is s_i .

For a circular foundation of width *B*, the coefficient α is a function of the ratio *H*/*B*. The thickness of the compressible soil is *H*, as given in Table 5.

In addition, Henkel (1956) established a correlation for two clayey soils to estimate the pore pressure coefficient (A) in the over-consolidation of two types of clays.

In this paper, for the studied case, the determination of μ , defined by Eq. (7) to correct the settlement estimation by the odometer method given by Eq. (8), is detailed in the following section.

The thickness of the compressible sublayers is H=2 m, and the foundation is a square rigid raft with $B_{sq}=15$ m. The equivalent diameter (B) of a circular raft of an area equal to the squared

Table 5. Values of coefficient α in Eq. (8) for a circular footing

H/B	α
0.00	1.00
0.25	0.67
0.50	0.50
1.00	0.38
2.00	0.30
4.00	0.28
10.0	0.26

rigid raft is

$$B = (4/\pi)^{0.5} B_{sq} \tag{9}$$

For $B_{sq} = 15$ m, the equivalent diameter is $D = 1.1287 \times 15 = 16.93$ m. Therefore, H/B = 0.1181. As given in Table 5, which uses a linear interpolation in the interval 0 < H/B < 0.25, $\alpha = 0.84$.

The studied Tunis soft clay is almost normally consolidated (Klai et al. 2015).

As given in Table 5, A = 0.95. Substituting the values of α and A into Eq. (8) gives $\mu = 0.95 + 0.84 \times (1 - 0.95) = 0.992 \approx 1$.

From Eq. (7), the correction coefficient proposed by Skempton and Bjerrum (1957) is approximately one, and the calculated values of the consolidation settlement using Terzaghi's 1D consolidation theory (which assumed zero horizontal displacement) are valid.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available from the corresponding author by request. This includes: the determination of the optimized ratio that includes the input and output data; calculations of the long-term settlement of the unreinforced soil and the time of consolidation; the prediction of settlement by the UCM that includes the input and output data.

Notation

The following symbols are used in this paper:

B = rigid raft width;

- c = cohesion;
- C_c = compression index;
- $c_v = \text{coefficient of vertical consolidation};$
- D_c = diameter of column;
- D_e = diameter of influence area;
- E = Young's modulus;
- $e_0 =$ initial void ratio;
- H = thickness of the soft deposit layers;
- $H_c =$ length of column;
- H_u = thickness of the unreinforced sublayer;
- N_c = number of columns;
- q = uniform vertical load;
- r_c = radius of column;
- r_e = radius of influence area;
- s =consolidation settlement;
- S_c = axis-to-axis spacing between columns;
- t_c = time of consolidation;
- $T_v = \text{time factor;}$
- U_v = degree of consolidation;
- z =layer depth;
- $\alpha \beta$ = coefficient, depth ratio; γ = unit weight;
- $\delta_{\rm LT} =$ long-term settlement;

 $\delta_{\rm res}$ = residual settlement;

- $\delta_{\rm ST} =$ short-term settlement;
- $\delta_{\text{tot,adm}}$ = allowable short-term settlement;
 - η = improvement area ratio;
 - $\Delta \sigma' = \text{excess of vertical stress};$
 - $\eta_{\rm opt}$ = optimized improvement area ratio;
 - σ'_{v0} = effective overburden stress;
 - v = Poisson's ratio; and
 - φ = drained friction angle.

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